Performance of Multi-Stratum Space-Time Coding for $N_r \times 2$ MIMO Channels

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***Abstract*—V-BLAST is a common architecture for transmitting multiple coded data streams over a MIMO channel. A drawback of V-BLAST, however, is that it offers little diversity. Multi-stratum space-time coding has been proposed as a generalization of V-BLAST, where the multiple transmitted data streams are first separately modulated using an orthogonal space-time block code and then are superimposed and transmitted over the multiple antennas. In this work, the performance of multi-stratum space-time coding for $N_r \times 2$ MIMO channels is analyzed, where the two superimposed streams are Alamouti modulated and coded at a rate optimized for the SNR. It is shown that the scheme offers significant improvement in terms of diversity while the encoding/decoding complexity remains essentially unchanged.**

***Index Terms*—MIMO, Alamouti code, multi-stratum space-time coding, fading, BLAST, diversity-multiplexing tradeoff.**

I. INTRODUCTION

CONSIDER a single-user multiple-input multiple-output (MIMO channel), where the transmitter and receiver are equipped with N_t and N_r antennas, respectively. The channel is given by,

$$\boldsymbol{y} = \sqrt{\rho}\mathbf{H}\boldsymbol{x} + \boldsymbol{n},$$

where $\mathbf{H}$ is an $N_r \times N_t$ matrix, $\boldsymbol{x}$ is the $N_t \times 1$ input vector and $\boldsymbol{y}$ is the $N_r \times 1$ received vector. The transmitter is subject to an average power constraint, $E[\|\boldsymbol{x}\|^2] \leq N_t$ where $\rho = \mathrm{SNR}$. We assume that the noise $\boldsymbol{n}$ is i.i.d. circularly symmetric complex Gaussian noise of unit power per antenna, i.e., $\mathbf{n} \sim \mathcal{CN}\left(0, I_{N_r \times N_r}\right)$. Thus, SNR is the maximal allowed average power per transmit antenna. We assume that the channel matrix $\mathbf{H}$ is fully known to the receiver but is not known to the transmitter (open loop). We further assume a quasi-static Rayleigh fading model where the channel is assumed constant throughout the duration of a codeword. The mutual information of the MIMO channel (in bits per channel use) for an i.i.d. Gaussian input is:

$$I_{\mathrm{OPT}}(\mathbf{H}) = \log_2 \left|1 + \rho\mathbf{H}\mathbf{H}^H\right|.$$

A well-known scheme that mainly focuses on maximizing the spatial multiplexing gain is the vertical Bell Labs space-time architecture (V-BLAST[1]). The input data is divided into independent substreams, which are separately coded and transmitted over different antennas. The receiver applies successive interference cancelation (SIC).

Multi-stratum space-time coding (MSSTC), introduced by Wachsmann et al. in [1], may be viewed as an enhanced V-BLAST-SIC scheme in which the data streams, after being separately coded, are then modulated via an orthogonal space-time block codes and then transmitted superimposed over the multiple antennas, whereby diversity is gained while the complexity remains essentially unchanged.

We analyze the performance of MSSTC for the case of a $N_r \times 2$ MIMO system where the code rates of the streams are optimized as a function of the SNR. We use the diversity-multiplexing tradeoff (DMT) framework, as defined in [2], as a means to compare different schemes. Let $R(\rho)$ (bits/symbol) be the rate of a code $C(\rho)$. A coding scheme $C(\rho)$ is said to achieve spatial multiplexing gain r and diversity gain d if the data rate satisfies $\lim_{\rho\to\infty} \frac{R(\rho)}{\log\rho} = r$ and the average error probability decays as $\lim_{\rho\to\infty} \frac{-\log P_e(\rho)}{\log\rho} = d$.

II. DMT OF ALAMOUTI MODULATION AND V-BLAST

We begin with recalling the DMT of Alamouti modulation and V-BLAST which will play an important role in the sequel.

Using Alamouti modulation [3] for the case of two receive antennas,[1] denote $\boldsymbol{y} = \begin{bmatrix} y_{1,1} & y_{1,2}^* & y_{2,1} & y_{2,2}^* \end{bmatrix}^T$ where $y_{i,j}$ is the symbol received at time j in antenna i; $\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$; $\boldsymbol{n} = \begin{bmatrix} n_{1,1} & n_{1,2}^* & n_{2,1} & n_{2,2}^* \end{bmatrix}^T$ where $n_{i,j}$ is the noise sample at time j in antenna i; and $h_{i,j}$ as the channel gain between transmit antenna j and receive antenna i.

With this notation, the received symbols can be written as

$$\boldsymbol{y} = \sqrt{\rho}\mathbf{H}_{\mathbf{eq}}\boldsymbol{x} + \boldsymbol{n}$$

where

$$\mathbf{H}_{\mathbf{eq}} = \begin{bmatrix} h_{1,1} & -h_{1,2}^* & h_{2,1} & -h_{2,2}^* \\ h_{1,2} & h_{1,1}^* & h_{2,2} & h_{2,1}^* \end{bmatrix}^T.$$

Linear processing at the receiver yields the equivalent scalar channel,

$$\tilde{y}_i = \sqrt{\rho} \cdot \|\mathbf{H}\|_{\mathrm{F}} \cdot x_i + \tilde{n}_i, \qquad i = 1, 2,$$

where F denotes Frobenius norm and where $\tilde{n}$ has the same distribution as n. The DMT curve for Alamouti modulation, depicted in Figure 1 (for a 2×2 system) is given by [2]

$$d(r) = 4(1 - r).$$

In [2], the DMT of several SIC decoding options for V-BLAST is analyzed. We outline the DMT curve for three variants of V-BLAST.

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[1]This scheme is sometimes referred to as H-BLAST [4].

[1]For notational convenience, the results are stated for the case of two receive antennas. The generalization to more receive antennas is straightforward.

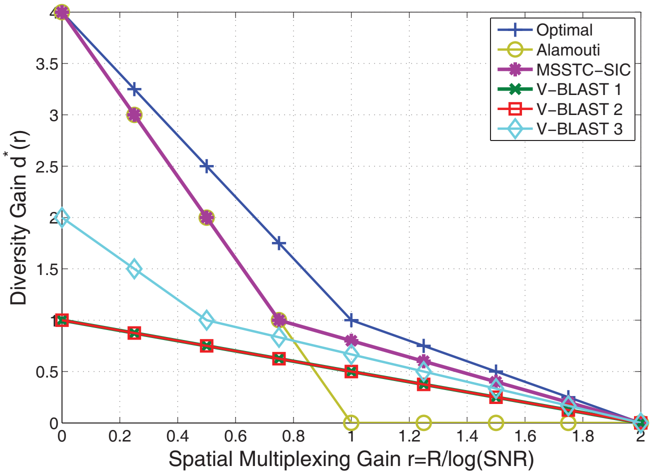

Fig. 1. Diversity-multiplexing tradeoff curves for 2×2 MIMO channel.

- "V-BLAST 1" - Allocate the same rate to both antennas, and arbitrarily decide on the order of decoding. The DMT curve of this scheme (for a square system with $N_t = N_r = n$) [2] is $d(r) = (1 - r/n)^+$.
- "V-BLAST 2" - It is shown in [2] that when the same data rate is sent over the antennas, the optimal decoding order is to choose the substream in each stage such that the SNR at the output of the corresponding decorrelator is maximized. The DMT curve of this scheme is upper bounded in [2] by $d(r) < (n - 1)(1 - r/n)$.
- "V-BLAST 3" - Fix the detection order but assign *different rates* to different substreams. For the 2x2 case, the optimal rate allocation is described as follows:
  - For $r \leq \frac{1}{2}$, only one substream is used. i.e., $r_2 = r$ and $r_1 = 0$. The DMT curve is thus $2(1 - r)$.
  - For $r > \frac{1}{2}$, two substreams are used where $r_2 = \frac{1}{2} + \frac{1}{3}\left(r - \frac{1}{2}\right)$ and $r_1 = \frac{2}{3}\left(r - \frac{1}{2}\right)$. The DMT curve is $1 + \frac{2}{3}\left(\frac{1}{2} - r\right)$.

We also define another V-BLAST variant which will be used for analysis of the proposed scheme. We analyze a scheme that decodes the *worst* antenna first. We denote this scheme by "V-BLAST 4" and in the sequel we prove that the DMT of this scheme is the same as for "V-BLAST 1".

III. MSSTC WITH OPTIMIZED RATE ALLOCATION

We now consider MSSTC with SIC being employed. It will be shown that the DMT curve of MSSTC-SIC is as depicted in Figure 1. For the case of two transmit antennas, MSSTC modulation is based on using two (different) embodiments of Alamouti modulation, and is described by

$$\tilde{\mathbf{X}} = \sqrt{P_1}\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} + \sqrt{P_2}\begin{bmatrix} x_3 & x_4^* \\ x_4 & -x_3^* \end{bmatrix}, \tag{1}$$

where each layer is coded with rate R_i (and power P_i). We also note that $P_1 + P_2 = \rho$, where for the moment we assume that $P_1 = P_2 = \frac{\rho}{2}$. Using the notation of Section II for $\boldsymbol{y}$ and $\boldsymbol{n}$ and denoting $\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$ we have

$$\boldsymbol{y} = \sqrt{\frac{\rho}{2}}\mathbf{H}_{\mathrm{MSSTC}} \cdot \boldsymbol{x} + \boldsymbol{n},$$

where

$$\mathbf{H}_{\mathrm{MSSTC}} = \begin{bmatrix} h_1 & h_2 & h_1 & h_2 \\ h_2^* & -h_1^* & -h_2^* & h_1^* \\ h_3 & h_4 & h_3 & h_4 \\ h_4^* & -h_3^* & -h_4^* & h_3^* \end{bmatrix}.$$

The resulting mutual information is

$$I_{\mathrm{OPT}}\left(\mathbf{H}_{\mathrm{MSSTC}}\right) = \frac{1}{2}\log_2\left|1 + \frac{\rho}{2}\mathbf{H}_{\mathrm{MSSTC}}\mathbf{H}_{\mathrm{MSSTC}}^H\right|.$$

IV. DMT OF MSSTC-SIC

We first compare the diversity of layer 2 in MSSTC with that of layer 2 (i.e., the stream transmitted over antenna 2) in "V-BLAST 3". For a 2×2 system, the channel matrix is given by

$$\mathbf{H}_{2\times 2} = \begin{bmatrix} h_1 & h_2 \\ h_3 & h_4 \end{bmatrix}.$$

For "V-BLAST 3", assuming correct SIC of antenna (layer) 1, the mutual information of antenna (layer) 2 is given by

$$C_{\mathrm{Ant2}}(\rho) = \log_2\left(1 + \rho(|h_2|^2 + |h_4|^2)\right).$$

According to [2], the outage probability for antenna 2 satisfies $P_e^{(2)}(\rho) \doteq \rho^{-2(1-r_2)}$ and thus $d_{\mathrm{Ant2}}(r_2) = 2(1 - r_2)$.

Assuming correct decoding of layer 1 in MSSTC, the mutual information of layer 2 can be written as

$$C_{\mathrm{Lay2}}(\rho) = \log_2\left(1 + \frac{\rho}{2}\sum_{i=1}^{4}|h_i|^2\right).$$

Since the modulation scheme of layer 2 is orthogonal, according to [2], the DMT curve of this layer is $d_{\mathrm{Lay2}}(r_{\mathrm{Lay2}}) = 4(1 - r_{\mathrm{Lay2}})$, which is significantly superior to the diversity of antenna 2 in "V-BLAST 3".

We now show that the diversity of layer 1 in MSSTC is the same as that of antenna 1 in "V-BLAST 3". Since SIC is information-lossless in Gaussian channels (see, e.g., [5]), the explicit expression for the mutual information of antenna 1 in "V-BLAST 3" is,

$$\begin{aligned} C_{\mathrm{Ant1}}(\rho) &= I_{\mathrm{OPT}}\left(\mathbf{H}_{2\times 2}\right) - C_{\mathrm{Ant2}}(\rho) \\ &= \log_2\left(\frac{1 + \rho^2\left|h_1h_4 - h_2h_3\right|^2 + \rho\sum_{i=1}^{4}|h_i|^2}{1 + \rho(|h_2|^2 + |h_4|^2)}\right). \end{aligned}$$

The outage probability of antenna 1 in "V-BLAST 3" satisfies [2] $P_e^{(1)}(\rho) \doteq \rho^{-(1-r_1)}$ and thus $d_{\mathrm{Ant1}}(r_1) = 1 - r_1$.

Explicitly writing the overall mutual information of MSSTC reveals that $I_{\mathrm{OPT}}\left(\mathbf{H}_{\mathrm{MSSTC}}\right) = I_{\mathrm{OPT}}\left(\mathbf{H}_{2\times 2}\right)$, i.e., there is *no loss* in overall capacity due to using MSSTC for the case of two transmit antennas. The explicit expression for the mutual information of layer 1 in MSSTC is therefore,

$$\begin{aligned} C_{\mathrm{Lay1}}(\rho) &= I_{\mathrm{OPT}}\left(\mathbf{H}_{2\times 2}\right) - C_{\mathrm{Lay2}}(\rho) \\ &= \log_2\left(\frac{1 + \rho^2\left|h_1h_4 - h_2h_3\right|^2 + \rho\sum_{i=1}^{4}|h_i|^2}{1 + \frac{\rho}{2}\sum_{i=1}^{4}|h_i|^2}\right). \end{aligned}$$

We next show that the diversity of layer 1 in MSSTC is equal to that of antenna 1 in "V-BLAST 3". We do so by showing that both are equal in turn to the diversity of antenna 1 in "V-BLAST 4".

Define $P_{\text{out}}^{\text{Arbitrary}}$ as the outage probability of the first antenna in "V-BLAST 1", $P_{\text{out}}^{\text{Best}}$ as the outage probability of the first antenna in "V-BLAST 2" and $P_{\text{out}}^{\text{Worst}}$ as the outage probability of the first antenna in "V-BLAST 4".

Since in "V-BLAST 1" the decoding order is chosen a priori, it follows from symmetry that the best and worst antenna are the first to be decoded with equal probability. Thus,

$$P_{\text{out}}^{\text{Arbitrary}} = \frac{1}{2} P_{\text{out}}^{\text{Best}} + \frac{1}{2} P_{\text{out}}^{\text{Worst}}$$

which in turn implies that,

$$P_{\text{out}}^{\text{Worst}} \mathrel{\dot{\leq}} 2P_{\text{out}}^{\text{Arbitrary}} \doteq P_{\text{out}}^{\text{Arbitrary}}.$$

Hence, the diversity of "V-BLAST 4" is the same as that of "V-BLAST 1" which in turn is given by $1 - r/2$.

Since in "V-BLAST 1" the DMT curve is dominated by that of antenna 1 and since the same rate is allocated to both antennas/layers, it follows that the DMT curve of the first antenna/layer is $d^{\text{V-BLAST 1}}(r) = 1 - r/2$ or equivalently $1 - r_1$. This is turn is equal to the DMT curve of the first antenna/layer in "V-BLAST 3".

We proceed to show that the same is true for layer 1 in MSSTC. We have,

$$C_{\text{Lay1}} \geq \log_2 \left(\frac{1 + \rho^2 \left| h_1 h_4 - h_2 h_3 \right|^2 + \rho \sum_{i=1}^{4} |h_i|^2}{1 + \rho \max(|h_1|^2 + |h_3|^2, |h_2|^2 + |h_4|^2)} \right)$$

The r.h.s. of the inequality above is $C_{\text{Ant1}}^{\text{Worst}}$ and thus

$$P_{\text{out}}^{\text{Lay1,MSSTC}} < P_{\text{out}}^{\text{Worst}} \doteq \text{SNR}^{-(1-r_1)}.$$

Hence $d_{\text{Lay1}}(r_1) = 1 - r_1$.

Now, using the argument used in [2] for expressing the DMT for V-BLAST 3 with replacing $d_{\text{Ant2}}(r_2) = 2(1 - r_2)$ with $d_{\text{Lay2}}(r_2) = 4(1 - r_2)$ we get:

$$d(r) = \max_{r_1, r_2} \left[\min_{i: r_i > 0} (1 - r_1), 4(1 - r_2) \right]$$

- For $r \leq \frac{3}{4}$, only one layer is used, i.e., $r_2 = r$ and $r_1 = 0$. The tradeoff curve is thus $4(1 - r)$.
- For $r > \frac{3}{4}$, the rates of the layer are chosen to equalize the diversity. The tradeoff curve is thus $\frac{8}{5} - \frac{4}{5}r$.

The resulting DMT curve is plotted in Figure 1. We observe that the latter is superior to the DMT of all three variants of V-BLAST. We also compare the outage capacities of all V-BLAST variants and MSSTC at 1% outage probability, as depicted in Figure 2.

V. MSSTC WITH EQUAL RATES

Thus far, we considered equal power allocation for the two layers while using different rates. Nonetheless, it is often desirable to allocate the same rate per antenna/layer so that a single code (encoder/decoder) can be used [4]. An advantage of MSSTC is that since the streams correspond to layers rather than different antennas, the power allocation can be freely chosen. Figure 3 depicts the outage capacity of MSSTC with equal rates. The power allocation as a function of SNR was numerically optimized.

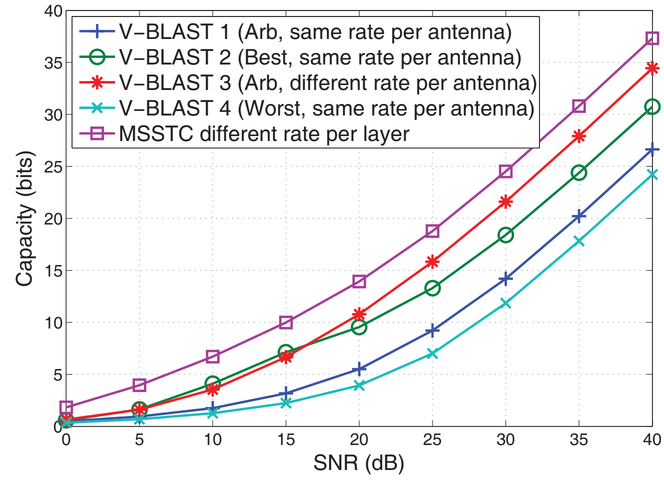

Fig. 2. 1% outage performance - 2×2 different rate per layer.

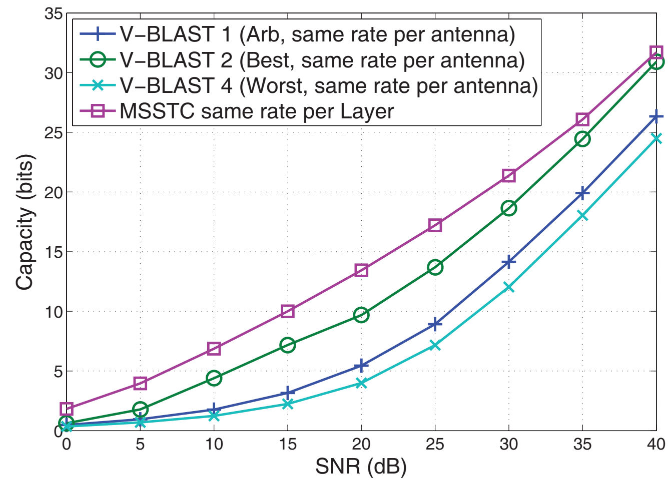

Fig. 3. 1% outage performance - 2×2 with same rate per layer.

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